ASYMPTOTIC FLOW OF A MONATOMIC DILUTE GAS IN NARROW GAP CHANNELS

P. A. Novikov and L. Ya. Lyubin

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The asymptotic flow of a dilute gas in gap channels is constructed within the ES-model. Flow with "slow" condensation (sublimation) on the channel flow is considered as an example.

The well-known quasidiffusive approximation [1] is used to analyze radiative or free-molecular transport in elongated channels. In [2-4] the corresponding approach was extended to transport problems in the narrow gap between parallel plates. Many technological processes occur, however, for finite values of the number Kn_H. In the present study we construct the asymptotic flow of a dilute gas in the gap between parallel plates, whose scale L in the gap symmetry plane is quite large in comparison with the molecular mean free path, so that the number Kn_H can vary within wide limits ($0 < Kn_H < \infty$). In the stationary case some constant field of macroscopic field parameters is established in the gap . If, in considering internal flows of a dilute gas, we confine ourselves to the linear approximation, corresponding to numbers $M \ll 1$ and a small relative temperature variation across the gap and at distances of order L in the symmetry plane, so that the square of these parameters can be neglected, then the basic kinetic equation of the ES-model [5] is written in the form

$$H\sqrt{h}\, v_i \frac{\partial f}{\partial x_i} = \alpha (f^+ - f). \tag{1}$$

In the linear approximation the integral of inverse collisions is

$$f^+ = f_0\left(1 - \frac{p_{ij}v_iv_j}{4pRT}\right),$$
$$p_{ij} = P_{ij} - \delta_{ij}p, \quad P_{ij} = m\int v_iv_jf d\bar{v} + O(M^2p), \quad p = \frac{p_{ii}}{3}. \tag{2}$$

Twice repeated subscripts imply summation.

If the flows are accompanied by transverse heat transport, the distribution function f has no symmetry with respect to the plane $X_3 = 0$. In the linear statement under consideration, however, this function can be represented by a perturbed absolutely Maxwellian distribution of the form

$$f = f_{00}(1 + \varphi_s + \varphi_a), \quad f_{00} = \frac{n_0}{(2\pi RT_0)^{3/2}} \exp\left(-\frac{v^2}{2RT_0}\right), \tag{3}$$

and the total problem can be decomposed into two independent ones for the symmetric φ_s and antisymmetric φ_a perturbations, respectively. We restrict ourselves to treating the symmetric problem only.

It is assumed that $H/L = \varepsilon \ll 1$. Therefore, as in the analysis [6] of continual flows of the Hill–Show type, it is advisable to introduce the dimensionless coordinates ξ_1, ξ_2, ζ, and specifically isolate the transverse velocity component $[\bar{v} = (\bar{u} + we_3)h_0^{-1/2}$, $h_0 = (2RT_0)^{-1}]$ and the derivative with respect to ζ. The local Maxwellian distribution f_0 appearing in the kinetic equation (1) can be replaced in the linear approximation by the expression

$$f_0 = f_{00}[1 + \nu + \left(u^2 + w^2 - \frac{3}{2}\right)\tau + 2\langle \bar{u}\rangle \bar{u} + 2\langle w\rangle w. \tag{4}$$

For simplicity it is assumed that the molecules reflected and emitted by the walls in a coordinate system fixed at the plates have a Maxwellian distribution corresponding to the wall temperature $T_w(\xi_1, \xi_2)$. Consider initially the auxiliary problem of a flow of a dilute

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gas in the region $-\infty < \xi_1, \xi_2 < \infty$, $|\zeta| \leq 0.5$, assuming that the Maxwellian distribution parameters of molecules flying from the wall n_w, T_w are given, i.e.,

$$f(\bar{r}, \zeta = \pm 0{,}5, w \lesseqgtr 0) = n_w(\bar{r}) \left[\frac{h_w(\bar{r})}{\pi}\right]^{3/2} \exp[-h_w(\bar{r}) v^2]. \tag{5}$$

The first and second order moments are sought of the distribution function, characterizing the gas flow at the arbitrary point $r_1 = r + \zeta e_3$ ($|\zeta| \leq 0.5$). As basis parameters of the absolutely Maxwellian distribution f_{00} we select $n_0 = n_w(\bar{r}_0)$, $T_0 = T_w(\bar{r}_0)$. If the perturbation φ_s is represented in the form

$$\varphi = \nu_w + (u^2 + w^2 - 1{,}5)\tau_w + \Phi, \tag{6}$$

then the following equation is obtained for the function Φ

$$\frac{w}{\alpha}\frac{\partial\Phi}{\partial\zeta} + \frac{\varepsilon}{\alpha}\bar{u}\nabla\Phi + \frac{\varepsilon}{\alpha}\bar{u}\left[\nabla\nu_w + \left(u^2 + w^2 - \frac{3}{2}\right)\nabla\tau_w\right] = -\Phi + \nu_0 + \left(u^2 + w^2 - \frac{3}{2}\right)\tau_0 + 2\langle\bar{u}\rangle\bar{u} + 2\langle w\rangle w - \left(\Pi_{11} - \frac{1}{2}\Pi_0\right)u_1^2 - \left(\Pi_{22} - \frac{1}{2}\Pi_0\right)u_2^2 - \left(\Pi_{33} - \frac{1}{2}\Pi_0\right)w^2 - 2\langle w\bar{u}\rangle w\bar{u} - 2\Pi_{12}u_1u_2 \tag{7}$$

and homogeneous boundary conditions

$$\Phi(\bar{r}, \zeta = \pm 0{,}5, w \lesseqgtr 1) = 0. \tag{8}$$

If one formally integrates the latter equation from the lower and upper walls, respectively, to the arbitrary point $u_1^{-1}d\xi_1 = u_2^{-1}d\xi_2 = \varepsilon w^{-1}d\zeta$, assuming that the moments appearing in it are known functions of coordinates, than, taking into account condition (8) at the walls, we obtain

$$\Phi(\bar{r}_1, w \gtreqless 0) = -\int_{\mp 0,5}^{\zeta} \Big\{\varepsilon\bar{u}\Big[\nabla\nu_w(\bar{r}') + \left(u^2 + w^2 - \frac{3}{2}\right) \times \nabla\tau_w(\bar{r}')\Big] - \alpha\Big[\nu_0(\bar{r}_1') + \left(u^2 + w^2 - \frac{3}{2}\right)\tau_0(\bar{r}_1') + 2\bar{u}\langle\bar{u}\rangle(\bar{r}_1') + 2w\langle w\rangle(\bar{r}_1') - u_1^2\left[\Pi_{11}(\bar{r}_1') - \frac{1}{2}\Pi_0(\bar{r}_1')\right] - u_2^2\left[\Pi_{22}(\bar{r}_1') - \frac{1}{2}\Pi_0(\bar{r}_1')\right] - w^2\left[\Pi_{33}(\bar{r}_1') - \frac{1}{2}\Pi_0(\bar{r}_1')\right] - 2w\bar{u}\langle w\bar{u}\rangle(\bar{r}_1') - 2u_1u_2\Pi_{12}(\bar{r}_1')\Big]\Big\}\exp\left[-\frac{\alpha}{w}(\zeta - \zeta')\right]\frac{d\zeta'}{w}. \tag{9}$$

If $\nu_w(\bar{r}')$ and $\tau_w(\bar{r}')$ are M times differentiable functions of their arguments, and, consequently, can be approximated by segment power series of the form

$$\nu_w(\bar{r}') = \sum_{i,j=0}^{i+j=M-1} \frac{\nu_{ij}(\xi_1' - \xi_1)^i(\xi_2' - \xi_2)^j}{i!\,j!}, \quad \nu_{ij} = \frac{\partial^{i+j}\nu_w(\bar{r})}{\partial\xi_1^i\,\partial\xi_2^j}, \tag{10}$$

then the asymptote of the flows considered is constructed as follows. Multiplying the last equation (9) by $\pi^{-3/2}\psi \exp(-V^2)$, where $\psi = 1$, $\bar{u}$, w, $w\bar{u}$, u_1^2, u_2^2, w^2, u_1u_2, and integrating the relations obtained over the whole space of dimensionless velocities $V = h_0 v$, we obtain a closed system of integral equations in the corresponding moments, which can, in turn, be represented in the form of asymptotic expansions in powers of the small parameters Kn_L

$$\nu_0 = Kn_L[N_\nu^{(1)}\nabla^2(\varepsilon\nu_w) + N_\tau^{(1)}\nabla^2(\varepsilon\tau_w)] + Kn_L^3[N_\nu^{(3)}\Delta^2(\varepsilon\nu_w) + N_\tau^{(3)}\Delta^2(\varepsilon\tau_w)] + O(Kn_L^5), \quad \langle\bar{u}\rangle = U_\nu^{(0)}\nabla(\varepsilon\nu_w) + U_\tau^{(0)}\nabla(\varepsilon\tau_w) + Kn_L^2[U_\nu^{(2)}\nabla\Delta(\varepsilon\nu_w) + U_\tau^{(2)}\nabla\Delta(\varepsilon\tau_w)] + O(Kn_L^4), \quad \langle W\rangle = Kn_L[W_\nu^{(1)}\Delta(\varepsilon\nu_w) + W_\tau^{(1)}\Delta(\varepsilon\tau_w)] + Kn_L^3[W_\nu^{(3)}\Delta^2(\varepsilon\nu_w) + W_\tau^{(3)}\Delta^2(\varepsilon\tau_w)] + O(Kn_L^5), \quad \langle wu\rangle = [\langle WU\rangle_\nu^{(0)}\nabla(\varepsilon\nu_w) + \langle WU\rangle_\tau^{(0)}\nabla(\varepsilon\tau_w)] + Kn_L^2[\langle WU\rangle_\nu^{(2)}\nabla\Delta(\varepsilon\nu_w) + \langle WU\rangle_\tau^{(2)}\nabla\Delta(\varepsilon\tau_w)] + \tag{11}$$

$$+ O(\mathrm{Kn}_L^4), \quad \Pi_{jj} = \mathrm{Kn}_L \left\{ \left[\Pi_{*\nu}^{(1)} \Delta + \Pi_{11\nu}^{(1)} \left(\frac{\partial^2}{\partial \xi_j^2} - \frac{\partial^2}{\partial \xi_i^2} \right) \right] (\varepsilon \nu_w) + \right.$$

$$\left. + \left[\Pi_{*\tau}^{(1)} \Delta + \Pi_{11\tau}^{(1)} \left(\frac{\partial^2}{\partial \xi_j^2} - \frac{\partial^2}{\partial \xi_i^2} \right) \right] (\varepsilon \tau_w) \right\} + O(\mathrm{Kn}_L^3) \quad \begin{pmatrix} i, j = 1, 2 \\ i \neq j \end{pmatrix},$$

$$\Pi_{33} = \mathrm{Kn}_L [\Pi_{33\nu}^{(1)} \Delta (\varepsilon \nu_w) + \Pi_{33\tau}^{(1)} \Delta (\varepsilon \tau_w)] + \mathrm{Kn}_L^3 [\Pi_{33\nu}^{(3)} \Delta^2 (\varepsilon \nu_w) +$$

$$+ \Pi_{33\tau}^{(3)} \Delta^2 (\varepsilon \tau_w)] + O(\mathrm{Kn}_L^5), \quad \tau_0 = \mathrm{Kn}_L [\theta_\nu^{(1)} \Delta (\varepsilon \tau_w) + \theta_\tau^{(1)} (\varepsilon \tau_w) +$$

$$+ O(\mathrm{Kn}_L^3), \quad \theta_{\nu,\tau}^{(1)} = \frac{8}{3} \Pi_{11\nu,\tau}^{(1)} + \frac{2}{3} \Pi_{33\nu,\tau}^{(1)} - N_{\nu,\tau}^{(1)}.$$

Here $\Delta = \nabla^2$, and the upper subscript $e = i + j - 1$, where i, j correspond to the expansion terms ν_{ij}, τ_{ij}. In constructing this asymptote one uses the power series expansions of the first and second order moments, appearing in the integrand expression of Eq. (9), relative to the point $\bar{r}_1$. For example,

$$\langle \bar{u} \rangle (\bar{r}' + \zeta' \bar{e}_3) = \langle \bar{u} \rangle (\bar{r} + \zeta' \bar{e}_3) + \sum_{i,j} \frac{\langle \bar{u} \rangle_{i,j} (\xi_1' - \xi_1)(\xi_2' - \xi_2)}{i!\, j!}.$$

The scalar coefficients denoted by capital letters are solutions of the following universal systems of integral equations (being independent of the temperature and pressure distributions in the gap):

$$U_\nu^{(0)}(\zeta) - \frac{\alpha}{\sqrt{\pi}} \int_{-0,5}^{0,5} U_\nu^{(0)}(\zeta') I_{-1}(\alpha |\zeta - \zeta'|) d\zeta' +$$

$$+ \frac{\alpha}{\sqrt{\pi}} \left(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5} \right) \langle WU \rangle (\zeta') I_0 (\alpha |\zeta - \zeta'| d\zeta' =$$

$$= -\frac{\alpha}{2\sqrt{\pi}} \int_{-0,5}^{0,5} I_{-1}(\alpha |\zeta - \zeta'|) d\zeta'; \quad \langle WU \rangle_\nu^{(0)} (\zeta) +$$

$$+ \frac{\alpha}{\sqrt{\pi}} \int_{-0,5}^{0,5} \langle WU \rangle_\nu^{(0)} (\zeta') I_1 (\alpha |\zeta - \zeta'|) d\zeta' - \frac{\alpha}{\sqrt{\pi}} \left(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5} \right) U_\nu^{(0)}(\zeta') \times$$

$$\times I_0 (\alpha |\zeta' - \zeta|) d\zeta' = -\frac{1}{2\sqrt{\pi}} \left(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5} \right) I_0 (\alpha |\zeta - \zeta'|) d\zeta',$$

$$N_\nu^{(1)}(\zeta) + \frac{\alpha}{\pi} \left(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5} \right) [3 U_\nu^{(0)}(\zeta') \alpha (\zeta - \zeta') I_{-2} (\alpha |\zeta - \zeta'|) -$$

$$- 2\sqrt{\pi} W_\nu^{(1)}(\zeta') I_0 (\alpha |\zeta - \zeta'|)] d\zeta' - \frac{\alpha}{\sqrt{\pi}} \int_{-0,5}^{0,5} \left\{ \left[\frac{3}{4} [N_\nu^{(1)}(\zeta') - \right.\right.$$

$$\left. - \theta_\nu^{(1)}(\zeta')] + \frac{1}{2} \Pi_{33\nu}^{(1)}(\zeta') + \frac{3}{\sqrt{\pi}} \alpha (\zeta - \zeta') \langle WU \rangle_\nu^{(0)}(\zeta') \right] I_{-1}(\alpha|\zeta - \zeta'|) + \tag{12}$$

$$\left. + \left[\frac{1}{2} N_\nu^{(1)}(\zeta') + \frac{3}{2} \theta_\nu^{(1)}(\zeta') - \Pi_{33\nu}^{(1)}(\zeta') \right] I_1 (\alpha |\zeta - \zeta'|) \right\} d\zeta' =$$

$$= \frac{3}{2\pi} \left(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5} \right) \alpha (\zeta - \zeta') I_{-2} (\alpha |\zeta - \zeta'|) d\zeta',$$

$$W_\nu^{(1)}(\zeta) + \frac{\alpha}{\pi} \int_{-0,5}^{0,5} [3 U_\nu^{(0)}(\zeta') \alpha (\zeta - \zeta') I_{-1} (\alpha |\zeta - \zeta'|) - 2\sqrt{\pi} W_\nu^{(1)}(\zeta') \times$$

$$\times I_1 (\alpha |\zeta - \zeta'|)] d\zeta' - \frac{\alpha}{\sqrt{\pi}} \left(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5} \right) \left\{ \left[\frac{3}{4} N_\nu^{(1)}(\zeta') - \frac{3}{4} \theta_\nu^{(1)}(\zeta') + \right. \right.$$

$$\left. + \frac{1}{2} \Pi_{33\nu}^{(1)}(\zeta') + \frac{3}{\sqrt{\pi}} \alpha (\zeta - \zeta') \langle WU \rangle_\nu^{(0)}(\zeta') \right] I_0 (\alpha |\zeta - \zeta'|) +$$

$$+\left[\frac{1}{2}N_\nu^{(1)}(\zeta')+\frac{3}{2}\theta_\nu^{(1)}(\zeta')-\Pi_{33\nu}^{(1)}(\zeta')\right]I_2(\alpha|\zeta-\zeta'|)\Big\}d\zeta'=$$

$$=\frac{3}{2\pi}\int_{-0,5}^{0,5}\alpha(\zeta-\zeta')I_{-1}(\alpha|\zeta-\zeta'|)d\zeta',$$

$$\Pi_{33\nu}^{(1)}(\zeta)+\frac{\alpha}{\pi}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)[3U_\nu^{(0)}(\zeta')\alpha(\zeta-\zeta')I_0(\alpha|\zeta-\zeta'|)-$$

$$-2\sqrt{\pi}W_\nu^{(1)}(\zeta')I_2(\alpha|\zeta-\zeta'|)]d\zeta'-\frac{\alpha}{\sqrt{\pi}}\int_{-0,5}^{0,5}\Big\{\Big[\frac{3}{4}N_\nu^{(1)}(\zeta')-$$

$$-\frac{3}{4}\theta_\nu^{(1)}(\zeta')+\frac{1}{2}\Pi_{33\nu}^{(1)}(\zeta')+\frac{3}{\sqrt{\pi}}\alpha(\zeta-\zeta')\langle WU\rangle_\nu^{(0)}(\zeta')\Big]I_{-1}(\alpha|\zeta-$$

$$-\zeta'|)+\left[\frac{1}{2}N_\nu^{(1)}(\zeta')+\frac{3}{2}\theta_\nu^{(1)}(\zeta')-\Pi_{33\nu}^{(1)}(\zeta')\right]I_3(\alpha|\zeta-\zeta'|\Big\}d\zeta'=$$

$$=\frac{3}{2\pi}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)\alpha(\zeta-\zeta')I_0(\alpha|\zeta-\zeta'|)d\zeta';$$

$$\theta_\nu^{(1)}=\frac{4}{3}\Pi_{*\nu}^{(1)}+\frac{2}{3}\Pi_{33\nu}^{(1)}-N_\nu^{(1)};$$

$$\Pi_{*\nu}^{(1)}(\zeta)+\frac{\alpha}{2\sqrt{\pi}}\int_{-0,5}^{0,5}\Big\{\Big[\Pi_{33\nu}^{(1)}(\zeta')-\frac{1}{2}N_\nu^{(1)}(\zeta')-\frac{3}{2}\theta_\nu^{(1)}(\zeta')\Big]I_1(\alpha|\zeta-\zeta'|)-$$

$$-\left[\Pi_{33\nu}^{(1)}(\zeta')+\frac{1}{2}N_\nu^{(1)}(\zeta')\right]I_{-1}(\alpha|\zeta-\zeta'|)\Big\}d\zeta'-$$

$$-\frac{\alpha}{\sqrt{\pi}}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)W_\nu^{(1)}(\zeta')I_0(\alpha|\zeta-\zeta'|)-$$

$$-\frac{3}{\sqrt{\pi}}U_\nu^{(0)}(\zeta')\alpha(\zeta-\zeta')I_{-2}(\alpha|\zeta-\zeta'|)\Big]d\zeta'=$$

$$=\frac{3}{2\pi}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)\alpha(\zeta-\zeta')I_{-2}(\alpha|\zeta-\zeta'|)d\zeta'-$$

$$-\frac{3\alpha}{2\pi}\int_{-0,5}^{0,5}\alpha(\zeta-\zeta')I_{-1}(\alpha|\zeta-\zeta'|)\zeta'd\zeta';$$

$$\Pi_{11\nu}^{(1)}(\zeta)+\frac{3\alpha}{2\pi}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)U_\nu^{(0)}(\zeta')\alpha(\zeta-\zeta')I_{-2}(\alpha|\zeta-\zeta'|)d\zeta'+$$

$$+\frac{\alpha}{2\sqrt{\pi}}\int_{-0,5}^{0,5}\left[\Pi_{11\nu}^{(1)}(\zeta')-\frac{3}{\sqrt{\pi}}\langle WU\rangle_\nu^{(0)}(\zeta')\alpha(\zeta-\zeta')\right]I_{-1}(\alpha|\zeta-$$

$$-\zeta'|)d\zeta'=\frac{3}{4\pi}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)\alpha(\zeta-\zeta')I_{-2}(\alpha|\zeta-\zeta'|)d\zeta';\quad \Pi_{12\nu}^{(1)}=2\Pi_{11\nu}^{(1)}.$$

$$U_\nu^{(2)}(\zeta)-\frac{\alpha}{\sqrt{\pi}}\int_{-0,5}^{0,5}\Big\{\Big[U_\nu^{(2)}(\zeta')-\frac{3\alpha}{\sqrt{\pi}}(\zeta-\zeta')W_\nu^{(1)}(\zeta')\Big]I_{-1}(\alpha|\zeta-\zeta'|)+$$

$$+\frac{27}{4\pi}U_\nu^{(0)}(\zeta'|\alpha^2(\zeta-\zeta')I_{-3}(\alpha|\zeta-\zeta'|)\Big\}d\zeta'+ \tag{13}$$

$$+\frac{\alpha}{\sqrt{\pi}}\left(\int_{-0,5}^{\zeta}-\int_{\zeta}^{0,5}\right)\Big\{\Big[\langle WU\rangle_\nu^{(2)}(\zeta')+\frac{3}{2\sqrt{\pi}}[2\Pi_{*\nu}^{(1)}(\zeta')-N_\nu^{(1)}(\zeta')]\times$$

$$\times\alpha(\zeta-\zeta')\Big]I_0(\alpha|\zeta-\zeta'|)+\Big[\frac{3}{\sqrt{\pi}}\Big[N_\nu^{(1)}(\zeta')-\Pi_{*\nu}^{(1)}(\zeta')-\frac{1}{2}\Pi_{11\nu}^{(1)}(\zeta')\Big]\times$$

$$\times\alpha(\zeta-\zeta')+\frac{27}{4\pi}\langle WU_\nu^{(0)}\rangle(\zeta')\alpha^2(\zeta-\zeta')\Big]I_{-2}(\alpha|\zeta-\zeta'|)\Big\}d\zeta'=$$

$$= -\frac{27}{8}\pi^{-3/2}\int_{-0,5}^{0,5}\alpha^2(\zeta-\zeta')^2 I_{-3}(\alpha|\zeta-\zeta'|)\,d\zeta';$$

$$\langle WU\rangle_\nu^{(2)}(\zeta) - \frac{\alpha}{\sqrt{\pi}}\Big(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5}\Big)\Big\{\Big[\Big[U_\nu^{(2)}(\zeta') - \frac{3\alpha}{\sqrt{\pi}}(\zeta-\zeta')W_\nu^{(1)}(\zeta')\Big]I_0(\alpha|\zeta-\zeta'|)\Big] +$$

$$+ \frac{27}{4\pi}U_\nu^{(0)}(\zeta')\alpha^2(\zeta-\zeta')^2 I_{-2}(\alpha|\zeta-\zeta'|)\Big\}d\zeta' +$$

$$+\frac{\alpha}{\sqrt{\pi}}\int_{-0,5}^{0,5}\Big\{\Big[\langle WU\rangle_\nu^{(2)}(\zeta') + \frac{3}{2\sqrt{\pi}}[2\Pi_{*\nu}^{(1)}(\zeta') - N_\nu^{(1)}(\zeta')]\alpha(\zeta- \tag{14}$$

$$-\zeta')\Big]I_1(\alpha|\zeta-\zeta'|) + \Big[\frac{3}{\sqrt{\pi}}\Big[N_\nu^{(1)}(\zeta') - \Pi_{*\nu}^{(1)}(\zeta') - \frac{1}{2}\Pi_{11\nu}^{(1)}(\zeta')\Big]\alpha(\zeta-\zeta') + \frac{27}{4\pi}\langle WU\rangle_\nu^{(0)}(\zeta')\times$$

$$\times\alpha^2(\zeta-\zeta')^2\Big]I_{-1}(\alpha|\zeta-\zeta'|)\Big\}d\zeta' = -\frac{27}{8}\pi^{3/2}\Big(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5}\Big)\alpha^2(\zeta-\zeta')^2 \times I_{-2}(\alpha|\zeta-\zeta'|)\,d\zeta',$$

$$I_m(x) = \int_0^\infty v^m \exp\Big(-v^2 - \frac{x}{v}\Big)dv.$$

The properties and tables of the integrals $I_m(x)$ are given in [7, 8]. Differentiating with respect to ζ the second equation (12), the second and third equations (13), the second equation (14), and using each time the available integrodifferential equations, we obtain the following relations:

$$\langle WU\rangle_\nu^{(0)} = -0{,}5\zeta,$$

$$\frac{\partial W_\nu^{(1)}}{\partial\zeta} = \frac{3\alpha}{2\pi}\int_{-0,5}^{0,5}[1-2\alpha U_\nu^{(0)}(\zeta')]I_{-1}(\alpha|\zeta-\zeta'|)\,d\zeta' - \frac{3\alpha^2}{2\pi}\times$$

$$\times\Big(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5}\Big)I_0(\alpha|\zeta-\zeta'|\zeta' d\zeta'; \quad \frac{\partial\Pi_{33\nu}^{(1)}}{\partial\zeta} = \frac{3\alpha}{2\pi}\Big(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5}\Big)[1-$$

$$-2\alpha U_\nu^{(0)}(\zeta')]I_0(\alpha|\zeta-\zeta'|)\,d\zeta' + \frac{3\alpha^2}{2\pi}\int_{-0,5}^{0,5}I_1(\alpha|\zeta-\zeta'|)\zeta' d\zeta',$$

$$\frac{\partial\langle WU\rangle_\nu^{(2)}}{\partial\zeta} = -\frac{27\alpha}{2\sqrt{\pi^3}}\Big(\int_{-0,5}^{\zeta} - \int_{\zeta}^{0,5}\Big)\Big\{[1-2\alpha U_\nu^{(0)}(\zeta')]\alpha(\zeta-\zeta')I_{-2}(\alpha|\zeta- \tag{15}$$

$$-\zeta'|) + \frac{4}{9}\sqrt{\pi}\,\alpha W_\nu^{(1)}(\zeta')I_0(\alpha|\zeta-\zeta'|)\Big\}d\zeta' -$$

$$-\frac{27\alpha^2}{2\sqrt{\pi^3}}\int_{-0,5}^{0,5}\Big\{\Big[\alpha(\zeta-\zeta')\langle WU\rangle_\nu^{(0)}(\zeta') + \frac{2}{9}\sqrt{\pi}\,[N_\nu^{(1)}(\zeta') -$$

$$-\Pi_{*\nu}^{(1)}(\zeta') - \frac{1}{2}\Pi_{11\nu}^{(1)}(\zeta')\Big]I_{-1}(\alpha|\zeta-\zeta'|) + \frac{\sqrt{\pi}}{9}[2\Pi_{*\nu}^{(1)}(\zeta') - N_\nu^{(1)}(\zeta')]I_1(\alpha|\zeta-\zeta'|\Big\}d\zeta'.$$

The functions $U_\nu^{(0)}$ and $\langle WU\rangle_\nu^{(0)}$ coincide with the corresponding parameters of the Poiseuille flow problem for the ES-model. Substitution of the first relation (15) into the first equation (12) shows that $U_\nu^{(0)} = U_c + U_{\nu 1}$, where U_c is the solution, found by various numerical methods [9-11], of the integral equation

$$U_c(\zeta) = \frac{1}{\sqrt{\pi}}\int_{-0,5}^{0,5}\Big[\alpha U_c(\zeta') - \frac{1}{2}\Big]I_{-1}(\alpha|\zeta-\zeta'|)\,d\zeta', \tag{16}$$

which corresponds to the Kruk model, while $U_{\nu 1}$ must satisfy the analog integral equation

$$U_{\nu 1}(\zeta) - \frac{\alpha}{\sqrt{\pi}}\int_{-0,5}^{0,5}U_{\nu 1}(\zeta')I_{-1}(\alpha|\zeta-\zeta'|)\,d\zeta' = F_2(\zeta). \tag{17}$$

The functions with subscript τ satisfy integral equations, differing from (12)-(15) only by the free terms. The moment expansion coefficients (11) are found as follows. Numerical solution of the integral equations (16) and (17) gives the function $U_\nu^{(0)}$ and the dimensionless velocity (discharge) $Q_\nu^{(0)}$ averaged over the height of the gap, the tangential stress $\langle WU\rangle_\nu^{(0)}$ is calculated by the first equation (15), while the proportional transverse velocity coefficient $W_\nu^{(1)}$ is obtained by integrating the right hand side of the second relationship (15) with the use of the symmetry condition $W_\nu^{(1)}(0) = 0$. The coefficients $N_\nu^{(1)}$, $\Pi_{33\nu}^{(1)}$, $\Pi_{*\nu}^{(1)}$ are found by simultaneous solution of the three corresponding equations (13) and using the functions already calculated $U_\nu^{(0)}$ and $W_\nu^{(1)}$, while the coefficient $\Pi_{11\nu}^{(1)}$ is found from the last equation (13). The coefficients $\langle WU\rangle_\nu^{(2)}$ and $U_\nu^{(2)}$ are found by subsequent solution of the integral equations (14) with the use of coefficients calculated earlier.

Several results of the calculation are given in Tables 1 and 2. The integral equations were solved by an iteration method.

The asymptote constructed above describes the flow of the dilute gas in the gap beyond the regions near the cylindrical surface elements blocking the gap or sealing it along the perimeter, near the open section of the gap and the discharge line of molecule emission intensity by the walls, i.e., the discharge line of the wall temperature T_w or the number density n_w of molecules flying from the wall. Boundary layers, which are not considered here, are formed in these specific regions.

The relations obtained above can be used to calculate flows outside these regions. Thus, in particular, one can treat flows between infinite plates, whose separate portions emit, absorb, or only reflect molecules, while the corresponding characteristics vary along the plates sufficiently smoothly or have weak jumps, capable only of generating uniformly small perturbations of the original parameters. The determination of the functions $\nu_w(\bar{r})$ and $\tau_w(\bar{r})$ is a major step in solving these problems.

We consider specifically the example of vapor flow in the gap, generated by sublimation or condensation at the walls. Using the commonly adopted scheme [12-15], based on the concept of a condensation coefficient β, characterizing the fraction of trapped surface phase transition molecules of the whole number contained in the flow incident on it, then one can write for the density of reflected surface molecules [14, 15]:

$$n_{rw} = n_{ew}(1-\beta)(1-\mu), \quad \mu = \frac{-2\sqrt{\pi}\, j}{\beta m n_{ew}\sqrt{2RT_w}}. \tag{18}$$

By (11) the vapor mass density flow toward the wall is

$$j = j_*\sqrt{2RT_w}\,, \quad j_* = n_w m\varepsilon\, \mathrm{Kn}_L (W_\nu^{(1)}\Delta\nu_w + W_\tau\Delta\tau_w)_{\zeta=0,5}. \tag{19}$$

The total number density of molecules flying from the walls is determined from the expressions [14, 15] $n_w = \beta n_{ew} + n_{rw} = n_{ew}[1 - \mu(1 - \beta)]$. For a constant density of vapor flow ($j = j_0 = \mathrm{const}$), corresponding to the approximation of a homogeneous distribution of thermal loads over the wall surfaces

$$n_w = n_{ew}(T_w) + \frac{2(1-\beta)\sqrt{\pi}\, j_0}{m\beta\sqrt{2RT_w}}. \tag{20}$$

By the Clapeyron–Clausius law

$$n_{ew}(T_w) = n_{ew}(T_0)\,\frac{T_0}{T_w}\exp\left[\frac{\Lambda}{R}\left(\frac{1}{T_0}-\frac{1}{T_w}\right)\right]. \tag{21}$$

For "slow" condensation (sublimation) ($|T_w - T_0| \ll T_0$) and by (1) expression (20) can be represented in linearized form:

$$n_w = n_w^0 - \chi(T_w - T_0),$$

$$n_w^0 = (1+\varkappa)\,n_{ew}^0 + \frac{1-\beta}{\beta}\,\frac{j_0}{m}\sqrt{\frac{2\pi}{RT_0}}\,; \qquad n_{ew}^0 = n_{ew}(T_0),$$

$$\chi = \frac{\varkappa n_{ew}^0}{T_0} + \frac{1-\beta}{\beta}\,\frac{j_0}{m}\sqrt{\frac{\pi}{2RT_0^3}}\,; \quad \varkappa = 1 - \frac{\Lambda}{RT_0}.$$

The function $W_{\nu,\tau}^{(1)}(0, 5, \alpha) = -Q_{\nu,\tau}^{(0)}(\alpha)(2\mathrm{Kn}_H)^{-1}$. Thus, following linearization we obtain from relation (19) and inhomogeneous Helmholtz equation, describing the plate temperature field for j = const:

TABLE 1. Dependence of the Functions $-U_\nu^{(0)}$ and $-Q_\nu^{(0)}$ on the Parameter α

α	$-U_\nu^{(0)}$, ξ = 0,0	0,1	0,2	0,3	0,4	0,5	$-Q_\nu^{(0)}$
0,01	1,5812	1,5753	1,5573	1,5252	1,4743	1,3800	1,5250
0,1	1,0853	1,0785	1,0575	1,0203	0,9616	0,8537	1,0203
0,2	0,9850	0,9773	0,9537	0,9120	0,8465	0,7268	0,9121
0,5	0,9144	0,9045	0,8741	0,8206	0,7374	0,5884	0,8214
1,1	0,9437	0,9297	0,8867	0,8119	0,6972	0,4975	0,8143
1,5	0,9913	0,9745	0,9235	0,8349	0,7001	0,4690	0,8383
2	1,0614	1,0412	0,9801	0,8745	0,7151	0,4460	0,8795
3	1,2178	1,1910	1,1097	0,9701	0,7619	0,4191	0,9785
5	1,5558	1,5153	1,3927	1,1844	0,8787	0,3932	1,2006
7	1,9065	1,8519	1,6874	1,4089	1,0049	0,3798	1,4337
10	2,4466	2,3705	2,1413	1,7555	1,4353	0,3686	1,7941

TABLE 2. Dependence of the Functions $-U_\tau^{(0)}$ and $-Q_\tau^{(0)}$ on the Parameter α

α	$-U_\tau^{(0)}$, ξ = 0,0	0,1	0,2	0,3	0,4	0,5	$-Q_\tau^{(0)}$
0,01	0,9357	0,9328	0,9236	0,9073	0,8815	0,8337	0,9072
0,1	0,7010	0,7061	0,6941	0,6728	0,6396	0,5798	0,6730
0,2	0,6786	0,6737	0,6588	0,6329	0,5925	0,5213	0,6333
0,5	0,6895	0,6822	0,6597	0,6209	0,5613	0,4596	0,6222
1,1	0,7809	0,76905	0,7328	0,6703	0,5761	0,4200	0,6736
1,5	0,8510	0,8361	0,7910	0,7134	0,5973	0,4075	0,7179
2	0,9409	0,9224	0,8665	0,7703	0,6277	0,3973	0,7766
3	1,1231	1,0976	1,0201	0,8879	0,6934	0,3849	0,8978
5	1,4895	1,4498	1,3295	1,1260	0,8296	0,3722	1,1437
7	1,8562	1,8021	1,5341	1,3637	0,9675	0,3651	1,3899
10	2,4093	2,3335	2,1050	1,7212	1,1717	0,3584	1,7611

$$\Delta T_w + B_1 T_w + B_2 T_0 = 0,$$

$$B_1 = \frac{2 j_* \overset{0}{n}_{ew}}{\overset{0}{n}_w \varepsilon^2 \varkappa_2 Q_\nu^{(0)}(\alpha)}, \quad j_* = \frac{j_0}{m \overset{0}{n}_{ew} \sqrt{2RT_0}}; \quad B_2 = \frac{1 - c_*}{c_*} B_1,$$

$$c_* = \frac{\varkappa_1}{\varkappa_2} - \frac{1}{2} + c_0, \quad c_0 = \frac{\beta\varkappa + (1-\beta)\sqrt{\pi}\, j_*}{\beta(1+\varkappa) + 2(1-\beta)\sqrt{\pi}\, j_*}, \tag{22}$$

$$\varkappa_1 = 1 - \gamma + c_0^2, \quad \varkappa_2 = 1 - \gamma - c_0, \quad \gamma = \frac{T_w \nabla P_w}{P_w \nabla \tau_w} = 1 - \frac{Q_\tau^{(0)}(\alpha)}{Q_\nu^{(0)}(\alpha)}.$$

In the two simplest cases, corresponding to a planar channel ($l = 0$) of length 2L open on both sides ($\xi_1 = \xi = \pm 1$) or a gap between parallel disks ($l = 1$) of radius $r_0 = L$ (open along the contour $\xi_1 = \xi = 1$), the operator

$$\Delta = \frac{d^2}{d\xi^2} + \frac{l}{\xi}\frac{d}{d\xi}, \tag{23}$$

and the plate temperature field, symmetric with respect to $\xi = 0$, is determined by the expression

$$T_w(\xi) = T_w(1) F_l(\xi, c_1) - \frac{1 - c_*}{c_*} T_0 [1 - F_l(\xi, c_1)],$$

$$F_0(\xi, c_1) = \frac{\cos(c_1 \cdot \xi)}{\cos(c_1)}; \quad F_1(\xi_1 c_1) = \frac{J_0(c_1 \cdot \xi)}{J_0(c_1)} \quad \text{for } B_1 > 0, \tag{24}$$

$$F_0(\xi, c_1) = \frac{\operatorname{ch}(c_1 \cdot \xi)}{\operatorname{ch}(c_1)}; \quad F_1(\xi, c_1) = \frac{I_0(c_1 \cdot \xi)}{I_0(c_1)} \quad \text{for } B_1 < 0.$$

Here $c_1 \sqrt{|B_1|}$, and $J_0(x)$ and $I_0(x)$ are the Bessel functions of real and imaginary arguments, respectively (they must be distinguished from the integrals $J_m(x)$ (14)).

Equation (24) makes it possible to estimate local surface heating by sublimation, as well as the possibilities of breakdown of material technological processes. To account for jumps of hydrodynamic parameters at the gap boundary one must investigate the "boundary layer" near the surface $\xi = 1$; for low intnesity j_* these jumps will be unimportant, while the variation of these parameters in the limit of a narrow ($\varepsilon \ll 1$) channel gap can be quite large.

NOTATION

λ, molecular mean free path; L, flow linear scale in the median gap plane; H, gap height; $Kn_L = \lambda/L$; $Kn_H = \lambda/H$; $\varepsilon = H/L$; f, distribution function; f_0 and f_{00}, local and absolute Maxwell distributions, $h = (2RT)^{-1}$; R, universal gas constant; T, local temperature; x_1, x_2, rectangular coordinates in the median plane; x_3, distance from median plane; n, molecular number density; m, molecular mass; $\bar{v} = v_1\bar{e}_1 + v_2\bar{e}_2 + v_3\bar{e}_3 = (\underline{u} + w\bar{e}_3)h^{-1/2} = \bar{V}h^{-1/2}$, vector of instantaneous molecular velocity; $\xi_j - x_j L^{-1}$ ($j = 1, 2$); $\zeta = x_3 H^{-1}$; $\bar{r} = \underline{\xi}_1 e_1 + \xi_2 e_2$, $\bar{r}_1 = \bar{r} + \zeta e_3$; $\nabla = \bar{e}_1(\partial/\partial\underline{\xi}_1) + \bar{e}_2(\partial/\partial\xi_2)$; $\Delta = \nabla^2$; $\nu = n - n_0/n_0$; $\tau = T - T_0/T_0$; e_j, unit vector of the x_j axis; $\alpha = \sqrt{\pi}H/3\lambda'$ $\nu_0 = \nu(\underline{r}_1) - \nu_w(\bar{r})$; $\tau_0(\bar{r}_1) = \tau(\underline{r}_1) - \tau_w(\bar{r})$; $\nu_0 = \langle 1\rangle_0$; $\Pi_0 = \nu_0 + \tau_0 = 2/3\langle u^2 + w^2\rangle_0$; $\Pi_{ij} = p_{ij}(\underline{r}_1')/p_w(r) - \delta_{ij}[1 + \Pi_w(r')]$; $\Pi_{ij} = \langle u_i u_j\rangle_0$; $\Pi_{3\gamma} = \langle wu_\gamma\rangle_0$; $\langle wu\rangle_0 = \Pi_{3\gamma}\underline{e}_\gamma$ ($\gamma = 1, 2$); $\langle\bar{u}\rangle = \langle u\rangle_0$, $\langle w\rangle = \langle w\rangle_0$; $\langle A\rangle_0 = \int Ae^{-u^2-w^2}\Phi dv$; $r' = r + \varepsilon(u/w)(\zeta - \zeta')$; $M = |\langle v\rangle| a^{-1}$; a, sound of speed; and $n_{ew}(T_w)$, equilibrium value of the molecular number density for wall temperature T_w (on the saturation line). Subscripts: w, at the wall; 0, at the point $\bar{r}_0$; and e_1, $\bar{e}_2$, e_3, unit vector corresponding to the axis x_1, x_2, x_3.

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